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STATISTICAL STUDIES OF THE PERFORMANCE OF COSMIC RAY RECORDING INSTRUMENTS

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Abstract

A method to determine the variance of statistical fluctuations for instruments used in the registration of the cosmic radiation - the neutron monitor, the duplex cubical counter telescope and the directional telescopes - is presented. The standard errors of data from cosmic ray instruments are estimated. The results are compared with the standard errors calculated from the theoretical Poisson distribution. It is stressed that the Poisson distribution will give an underestimation of the standrad errors.

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System of notations

The following system of notations is used:

In the population:

$D(x)$	The standard deviation of the variable x
$D^2(x)$	The variance of the variable x
$E(x)$	The mean of the variable x
$C(xy)$	The covariance of the variables x and y.

In the sample:

s^2	The variance calculated from the sample
$S(x)$	The standard error of the variable x
$\bar{x}$	The mean of the variable x
$\wedge$	indicates estimation

I. Introduction

In dealing with investigations where counting tubes are used, it is common to assume that each observation is drawn from a Poisson distribution. It is well-known that a series of random events will give the probability that n events will occur in the interval t (for instance Curran & Craggs, 1949).

$$P(X = n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \tag{1}$$

where λ is the average number of events per second. As long as the events will occur at random, which holds for most radioactive processes and for cosmic radiation, the Poisson distribution will be theoretically exact. This problem has been theoretically discussed by several authors (Bateman 1910 and Fry 1928). In connection with Bateman's paper of 1910, Rutherford & Geiger (1910) made their famous experiment on the distribution of α-particles from a radioactive sample. The result showed good agreement with the Poisson distribution.

This gives the variance of the counting rate N

$$D^2 (X) = N \tag{2}$$

The discrete Poisson distribution called "the law of small numbers", is skew for small values of λt but when increasing λt the distribution forms more symmetrical and will soon be approximately equal to the continuous normal distribution. According to Wolfenden (1941) the skewness of the distribution will be negligible for

$$\lambda t = N \geqq 10$$

In registrations of cosmic radiation the use of counting tubes are common and for error calculations, the Poisson distribution is of basic importance. One must be aware that this estimate will only take care of the true random variations in the number of particles. It is sometimes used uncritically, regardless of the existence of other types of errors which may be introduced by the instruments and during the data processing.

In this paper the standard errors of cosmic ray data from different instruments are calculated and the results are discussed and compared with expected theoretical values.

II. Statistical treatment

There are difficulties in error calculations of time series analyses. The general formula for the estimation of the variance is:

$$\hat{D}^2(x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

If the variable x is a function of time, the result will be an estimate including both the time variations of the variable and the statistical random fluctuations. Thus this method will be unsuitable for our purpose.

Mc Cracken (1958) has chosen a very calm cosmic radiation period for such a calculation and got a fairly good estimate of the variance $\hat{D}^2(x)$ due to only statistical fluctuations, although the existence of a daily variation and a possible 27-day variation will result in overestimation of $D^2(x)$.

To pass these problems there opens a possibility by the IGY recommendations to divide the cosmic ray instruments in equal sections to get an unbroken continuous registration. By serving each identical section of the instrument through a complete set of electronics a good defence against an all-over breakdown is offered. Each section is working to some extent as an independent instrument. Only the power line is common. The sections are measuring the same phenomena. By calculating the differences or the ratios between those two sections we have a method to get the statistical random fluctuations.

Each observed value can be written as

$$N = E(I) + f_1(P,T) + f_2(\Delta I) + \varepsilon \tag{3}$$

where $E(I)$ = the true mean of the intensity of the cosmic radiation during the period of observation

$f_1(P,T)$ = the atmospheric influence of the cosmic radiation where the pressure P and the temperature T are functions of time.

$f_2(\Delta I)$ = the time variation of the primary radiation. We assume that the variation at the top of the atmosphere is proportional to the variation at the sea level for constant atmospheric parameters (Ehmert).

ε = random variable due to statistical fluctuations, which will consist of both the Poisson error and the instrumental errors.

We know that $E(\varepsilon) = 0$

There exist different theories of the atmospheric influence of the radiation (Dorman, Duperier etc.) but for this calculation it is sufficient to use the simple linear form of the wellknown Duperier's formula

$$dN = \alpha\, dP + \beta\, dT + \gamma\, dH \tag{4}$$

dN is the difference between the observation and the corrected value; dP, dT and dH are the differences between the means and the observed values of pressure, temperature and height respectively, to a certain pressure level (100 or 200 mb); α, β and γ are constants.

Our knowledge of the time variations of the primary radiation is limited. We know to some extent the periodical variations such as those with periods of 24 hours and 27 days but the nonperiodicial ones, i.e. Forbush decreases, are still uncertain.

The two identically built sections record the same radiation. We introduce the ratio between the true counting rates:

$$k = \frac{I_{x_1}}{I_{y_1}} = \frac{E(I_x)}{E(I_y)} \tag{5}$$

where I_x and I_y are the number of counts in the resp. sections due to cosmic radiation when the atmosphere is supposed to be constant from time to time.

z is the number of counts recorded in one section that due to multiplicity in the instrument will be detected in the adjacent section simultaneously.

The time variables P, T, H, z and I_y are not true random. For a longer period the distributions of the variables tend to be symetrical around the mean. P, T and I_y are independent and we assume that their covariances will be negligible. This will not be too misinterpreted, especially as we in the follwoing are using the differences of (x-y), which will reduce the influence of existing covariance term. But

$$z = f_1(I_y)$$
$$\Delta H = f_2(\Delta P)$$

In the following the symbols D, E and C will mean the variance, mean and covariance independent of the variables are true random or not.

We know

$E(\Delta P) = E(\Delta T) = E(\Delta H) = 0$

The registrated values from each section are divided by using eg. (3), (4) and (5) for a certain time

$$x_1 = I_{x_1} + z_1 + (I_{x_1} + z_1)(\alpha\Delta P_1 + \beta\Delta T_1 + \gamma\Delta H_1) + \varepsilon_x =$$

$$= kI_{y_1} + z_1 + (kI_{y_1} + z_1)(\alpha\Delta P_1 + \beta\Delta T_1 + \gamma\Delta H_1) + \varepsilon_x$$

$$y_1 = I_{y_1} + z_1 + (I_{y_1} + z_1)(\alpha\Delta P_1 + \beta\Delta T_1 + \gamma\Delta H_1) + \varepsilon_y$$

we have:

$$E(x) = kE(I_y) + E(z) \tag{6}$$

$$E(y) = E(I_y) + E(z) \tag{7}$$

We form:

$$A = \alpha^2 D^2(P) + \beta^2 D^2(I) + \gamma^2 D^2(H) + 2\alpha\gamma C(PH)$$

and

$$D^2(x) = k^2D^2(I_y) + D^2(z) + A\left\{k^2E^2(I_y) + E^2(z) + k^2D^2(I_y) + D^2(z)\right\} + 2k\,C\,(I_{y_1}z)\left\{1+A\right\}$$
$$+ 2kD^2(P)E(I_y)\,E(z) + D^2(\varepsilon_x) \tag{8}$$

$$D^2(y) = D^2(I_y) + D^2(z) + A\left\{E^2(I_y) + E^2(z) + D^2(I_y) + D^2(z)\right\} + 2C(I_{y_1}z)\left\{1+A\right\}$$
$$+ 2\,D^2(P)\,E(I_y)\,E(z) + D^2(\varepsilon_y) \tag{9}$$

$$C(xy) = kD^2(I_y) + D^2(z) + A\left\{kE^2(I_y) + E^2(z) + kD^2(I_y) + D^2(z)\right\} +$$
$$\left[C(I_y z)\left\{1+A\right\} + D^2(P)E\,(I_y)\,E(z)\right](k+1) \tag{10}$$

<u>Difference:</u> The differences (x-y) give from eq. (6) and (7):

$$E(x-y) = (k-1)\,E(I_y) = M$$

$$D^2(x-y) = D^2(x) + D^2(y) - 2\,C(xy)$$

where

$$C(xy) \neq 0$$

as x and y are correlated. Eq. (8), (9) and (10) give:

$$D^2(x-y) = (k-1)^2\left[D^2(I_y) + \left\{E^2(I_y) + D^2(I_y)\right\}A\right] + D^2(\varepsilon_x) + D^2(\varepsilon_y) \tag{11}$$

where $(k-1)^2 = \dfrac{\left\{E(I_x) - E(I_y)\right\}^2}{E^2(I_y)} = \dfrac{M^2}{E^2(I_y)}$

We are interested in the distributions of ε_x and ε_y, which describe the true random fluctuations.

Ratio: We form

$$\frac{x_1}{y_1} = \frac{kI_{y1} + z_1 + (kI_{y1}+z_1)(\alpha \Delta P_1 + \beta \Delta T_1 + \gamma \Delta H_1) + \varepsilon_x}{I_{y_1} + z_1 + (I_{y_1} + z_1)(\alpha \Delta P_1 + \beta \Delta T_1 + \gamma \Delta H_1) + \varepsilon_y}$$

The variance of a function $F(x_1\ x_2\ x_3 - - -)$ can be formed (Kendall)

$$D^2(F) = \sum \left\{ \left(\frac{\partial F}{\partial x_i}\right)^2 D^2(x_i) \right\} + \sum \left\{ \frac{\partial F}{\partial x_i} \frac{\partial F}{\partial x_k}\ C\ (x_i\ x_k) \right\}$$

when $i \neq k$

$$D^2\left(\frac{x}{y}\right) = \frac{1}{E^2(y)} D^2(x) + \frac{E^2(x)}{E^4(y)} D^2(y) - 2\frac{E(x)}{E(y)E^2(y)} \left[E(xy) - E(x)\ E(y)\right]$$

If I_y and $kI_y \gg z$ and $k \approx 1$ we can write:

$$D^2\left(\frac{x}{y}\right) \doteq \frac{(k-1)^2}{E^2(I_y)} \left[D^2(z) + A\left\{E^2(z) + D^2(z)\right\}\right] + \frac{1}{E^2(I_y)} \left[D^2(\varepsilon_x) + k^2 D^2(\varepsilon_y)\right] \quad (12)$$

If we assume $z = D^2(z) = 0$

$$D^2\left(\frac{x}{y}\right) \doteq \frac{1}{E^2(I_y)} \left[D^2(\varepsilon_x) + k^2 D^2(\varepsilon_y)\right] \quad (13)$$

Remarks: Due to eq. (11) the variance of (x y) is dependent of k, the atmospheric effects and the variations of the primary radiation.

The distribution of the ratio can be assumed approximately normal under the following conditions:

$E(I_x)$ and $E(I_y) \gg 0$

I_x and $I_y \gg \varepsilon_x$ and ε_y respectively.

This is fulfilled in most cases when we are dealing with observations of cosmic radiation. The variance is due to eq. (12) independent of atmospheric and cosmical variations for $z = 0$.

III. Discussion of errors introduced by the method of registration

In registration of cosmic radiation it is often useful to scale the data before the recording. The registered value is sometimes rounded off to make the following data processing as simple as possible. The factor used in scaling and rounding off is chosen according to the counting rate. We shall here discuss the errors introduced by the methods.

Scaling: Scaling can be made in different ways but at continuous registration it is a rule to scale without zero resetting. This means that all counts arrived in the scaler and not scaled is carried over to next period of registration.

We assume at the time of registration a fraction of m, the scaling factor, consisting of N_e counts has arrived in the scaler and will not be registered. The following registration will then start with N_f counts. For one registration we write

$$x_1 = mN_1 + N_f - N_e$$

where mN_1 is the number of counts, which are registered as scaled. N_e and N_f can be assumed as independent. We form

$$E(x_1) = mE(N_1) + E(N_f) - E(N_e) \quad (14)$$

$$D^2(x_i) = D^2(N_i) + \bar{r}^2 (N_e + N_f) + 2\ mC\ \left[N_i, (N_e + N_f)\right] \tag{15}$$

To draw conclusions of the corrections, which must be applied to the variance of the scaled distribution to get the true distribution, is rather complicated. The influence of the covariance term is dependent of the shape of the distribution, the magnitude of the scaling factor and how the group intervals are situated in the distribution. As long as the scaling unit is chosen small compared with the standard deviation of the distribution of x the influence can be negligible. In this paper all observations are scaled with a small unit and no corrections are made.

The distributions of N_e and N_f are discrete with the range (0,m). If the distribution of x is approximately symetrical, which is the case when the period of observation is long, we can assume that

$$E(N_e) = E(N_f).$$

Thus the method will not introduce any systematical error according to eq. (14).

Rounding off. When tabulating data it is often useable to round off the data to simplify the follwoing data process. Eq. (14) will then be formed when q is the rounding off factor

$$E(x_i) = qE(N_i) + E(N_e) \tag{16}$$

The method of rounding off is chosen that $E(N_e)$ is as close to zero as possible. One method of rounding off often used is, that all values ended with

$$\frac{N_e}{q} \geq \frac{1}{2}$$

are increased with one unit. $E(N_e)$ is not exactly zero but the difference is small. The distribution of N_e is discrete with the range

$$\left(-\frac{q}{2}, +\frac{q}{2}\right)$$

In this paper no rounded off values are used.

Influence of the recording time. As yet we have assumed the time of the recording as zero. It is desirable to hold this time as short as possible. Photographic methods are then excellent but mechanical printing counters are difficult to build with short printing time. During the time of recording process there is a probability that a counts has arrived in the scaler. The mechanical counter is not open for counts during printing and a unit is missed. This will result, that $E(N_f) < E(N_e)$ in eq. (14) and a systematic error is introduced. As a rule the time of printing is small e.g. the mechanical counters used for the counter telescopes at Uppsala and Murchison Bay have a printing time of 0.8 sec. For the international cube, which has a counting rate of each section of about 5×10^4 counts/hour, every tenth recording will miss one unit. The introduced systematical error will be of the magnitude of 5 per cent of m.

IV Results:

Neutron monitor: In order to get an estimation of the standard deviation of the true statistical fluctuations for an IGY standard neutron monitor (Simpson 1953) data from three different stations have been used.

	lat.	long.	Altitude	Mean counting rate per hour
Murchison Bay	$80^{\circ}03'$N	$18^{\circ}15'$E	Sea level	26000
Uppsala	$59^{\circ}51'$N	$17^{\circ}55'$E	Sea level	25000
Mt. Wellington	$42^{\circ}55'$S	$147^{\circ}14'$E	725 m	18000

The stations are all equipped with neutron monitors built after the IGY recommendations in two sections, each served by a complete set of electronics. Great care has been taken to use such periods for the calculation, when the two sections of the instruments have been working without change of counting. Diagram 1 shows $\hat{M}$ for 10-day periods from the three stations during the periods used in the calculations.

The differences between the two sections are calculated for one-hour values uncorrected for atmospheric effects. We then have the sample mean and variance by the usual statistical formulae:

$$\bar{d} = \frac{1}{n} \sum (x-y)$$

$$s^2(x-y) = \frac{1}{n} \sum (d-\bar{d})^2$$

where d is the difference. We estimate

$$\bar{d} = E\ (x-y)$$

$$s^2(x-y) = \frac{n-1}{n} D^2\ (x-y)$$

For large values of n we find that S^2 is a good estimate of D^2. This happens in most cases in this paper.

The standard errors are calculated from

$$s(\bar{d}) = \frac{s}{\sqrt{n}} \tag{17}$$

$$s(s^2) = \frac{s^2 \sqrt{2n}}{n} \tag{18}$$

$$s(s) = \frac{s\sqrt{2n}}{2n} \tag{19}$$

These equations are valid when the variable is drawn from a normal distribution. Diagram (2) shows the distribution of (x-y) for Murchison Bay with the normal distribution fitted. As seen the distribution can be assumed approximately normal.

Eq. (11) shows that calculating $D^2(x-y)$ the result will be an overestimate of

$$D^2(\varepsilon_x) + D^2(\varepsilon_y)$$

for $M \neq 0$. We can calculate the degree of overestimation if we know M, $E(I_y)$, $D^2(I_y)$, $D^2(P)$, $D^2(T)$, $D^2(H)$, α, β, and γ.

For the neutron monitor:

$$\beta \simeq \gamma \simeq 0$$

$D^2(P)$ and $D^2(I_y)$ are more difficult to estimate. Meteorological records give information of $D^2(P)$. For Stockholm, which is a typical sea level station at a middle-high latitude, a mean covering 10 years gives

$$\hat{D}\ (P) = 12 \text{ mb}$$

This value can be expected to increase during summer and decrease during winter. The variations are small.

$D^2(I_y)$ is the variance of the primary radiation and will vary greatly according to whether the period choosen for the calculation is cosmically calm or not and at which latitude the station is situated. Diagram 3 is drawn for the stations at Uppsala and Murchison Bay for different values of $D^2(I_y)$ and $D^2(P)$. Following assumptions are made:

$E(I_y) = 12500$ counts/hour

$\alpha = 0.73$ %/mb

$D(I_y) = 100, 500, 1000$ counts/hour

$D(P) = 9, 12, 15$ mb

From the diagram 3 it is easily shown that for small values of M the overestimation will be negligible. If we can accept an overestimate of 1 per cent of

$$D^2(\varepsilon_x) + D^2(\varepsilon_y)$$

(for Uppsala and Murchison Bay ~ 400 counts/hour) we can use $M \leq 200$ counts/hour.

Table number 1 shows $D^2(x-y)$ for separate neutron monitors. The result shows poor agreement with the theoretical value given by the Poisson distribution. According to Mc Cracken (1958) this is due to multiplicity of the particle production in the atmosphere as well as to star production in the monitor itself. He has shown theoretically how the multiplicity gives a higher value of the standard deviation than expected from the Poisson distribution. He used experimental values from a small two-counter monitor and found the standard deviation of the distribution:

$$\hat{D}(N) = 1.13\ \sqrt{N}$$

where N is the counting rate. He made a rough estimate for a standard I.G.Y. monitor consiting of 12 counters

$$\hat{D}(N) \approx 1.2\ \sqrt{N}$$

A mean covering three periods from Murchison Bay and Uppsala which have monitors of similar appearance and electronics give

$$\hat{D}(N) = 1.181\ \sqrt{N}$$

The result shows good agreement with the one of Mc Cracken. The result from Mt. Wellington can not be immediately compared with this value as $\hat{M} = 460$ counts/hour.

We can then use the ratio between one hour observations of the two sections to find the distribution of ε_x and ε_y. By calculating $D^2(\frac{x}{y})$ we get an overestimate of

$$D^2(\varepsilon_x) + x_k\, D^2(\varepsilon_y)$$

from eq. (12) due to z. Now we know that z is only a small fraction of the counting rate according to Mc Cracken (1958).

We assume:

$z \approx 4$ per cent of the counting rate

$D(P) \approx 10$ mb

$\alpha \approx 0.7$ per cent per mb

$D(z) \approx D(I_y)$

which give a negligible overestimate. Eq. (13) can be used.

Table 1 gives the result from Mt. Wellington for the calculation of the ratio

$$\hat{D}(N) = (1.25 \pm 0.02)\ \sqrt{N}$$

This value is markedly higher than those from Uppsala and Murchison Bay. The data of Mt. Wellington are scaled with a factor 64 while the Uppsala and Murchison Bay data are unscaled. The correction of the "scaled" variance to get the "true" variance is difficult to find as seen in a preceding section. However, the correction is small. Shepeard's correction for the variance $-1/12\ m^2$ (see Cramer 1945) gives a rough estimate.

The difference between the calculated variances indicates that D(N) is an instrumental constant. The method used to estimate the variance, will include all statistical errors introduced by the method of registration, the Poisson error as well as the instrumental errors. The neutron monitors may differ in appearance and electronics, which can cause a variability of D(N) from one station to another and of course also from time to time according to the condition of the instrument.

Counter telescope. The variance of the statistical fluctuations for the duplex cubical counter telescope and the directional telescope are calculated by the same method as in the section of the neutron monitor. 2-hour values uncorrected for atmospheric effects from the two stations at Uppsala and Murchison Bay, which are equipped with a duplex cubical counter telescope built after I.G.Y. recommendations and a telescope for east and west directions, are used. Descriptions of the instruments are found elsewhere (Sandström, Dyring, Lindgren 1960). As in the calculation of the neutron monitor great care is here taken to chose periods of observations where no changes in the counting rate occur. It is sometimes difficult to find such reliable periods for counter telescopes due to their construction.

Duplex cubical counter telescope. Fig. 4 shows the daily mean intensity during the periods used in the calculation for the international cube. If there exists a difference between the counting rates of the two sections $D^2(x-y)$ there is an overestimation of the sum of $D^2(\varepsilon_x)$ and $D^2(\varepsilon_y)$ acc. to eq. (11). For the duplex cubical counter telescope we assume:

α = 0.15 per cent/mb

β = 0.1 per cent/°C (200 mb level)

γ = 6 per cent/km (100 mb level)

D(P) = 10 mb

D(T) = 10°C (200 mb level)

D(H) = 0.5 km (100 mb level)

Correlation coeff. (PH) = 0.5

$E(I_y) = 10^5$ counts/ 2 hour

$D(I_y) = 5000$ counts/2 hour

These values are roughly estimated. For a maximal overestimation of 1 per cent we can allow

$\hat{M} \leq 900$ counts/ 2 hour

The result of the calculations are shown in table 2. The mean over all periods for each station gives

Uppdala $\hat{D}(N) = 1.118\ \sqrt{\bar{N}}$

Murchison Bay $\hat{D}(N) = 1.277\ \sqrt{\bar{N}}$

Directional telescope. The instruments at Uppsala and Murchison Bay have eight channels, four in each direction. The channels directed east have odd and the west one even numbers. Figs. 5, 6, and 7 show the daily mean intensity during the periods used in the calculations. We make roughly the same assumptions as for the international cube a part from

$D(I_y) = 3\cdot 10^4$ counts/2 hour

D(N) = 1500 counts/ 2 hour

Then we may allow for 2-hour values

$\hat{M} \leq 500$ counts

for a maximum overestimation of 1 per cent. The four channels of each direction give six sets of differences. Only two are independent. Thus we have six equations and four variables $\mathcal{E}_x$, $\mathcal{E}_y$, $\mathcal{E}_z$, $\mathcal{E}_v$. By the common least square method the variables are estimated. The results are shown in tables 3 and 4. The mean over all periods for each station give

$$\text{Uppsala} \quad \hat{D}(N) = 1.102 \sqrt{N}$$
$$\text{Murchison Bay} \quad \hat{D}(N) = 1.167 \sqrt{N}$$

Remarks. The influence of the multiplicity (Mc Cracken 1958) is smaller for a counter telescope than for a neutron monitor. The time dealy between mesons produced by the same primary particle, which can be detected in each counter telescope section, will mostly be small. The resolving time is not sufficient to separate particles derived from the same primary. The local production of mesons is also small. Thus we can expect a closer agreement to the Poisson distribution for a counter telescope than for a neutron monitor. Still the results do not indicate this and we find further a difference in the variance of ε from Uppsala and Murchison Bay and between different channels of the same instrument as well as from period to period. This indicates that the standard deviation of ε is an instrumental constant. The counter telescope, which is equipped with GM-counters, univibrators, sharpeners, coincidence circuits and scalers, introduces errors due to resolving time, spurious counts etc., and are comparatively sensitive to changes in the power line and trimming conditions. These types of errors are more prominent on a counter telescope than on the neutron monitor.

The station of Murchison Bay was situated in an area with hard weather conditions and its electric source was diesel-engined generators. Compared to the more quiet conditions of Uppsala it is not surprising to find a higher variance of ϵ for Murchison Bay.

It must be stressed that the periods of calculations are very carefully chosen. As a rule one is forced to accept data with less accuracy to get continous registrations. Thus it is obvious that the Poisson distribution a counter telescopes gives an underestimation in calculating the standard errors of the data.

Pressure of the atmosphere. To correct the cosmic ray data for atmospheric effects it is necessary to have continuous registration of the atmospheric pressure. At the cosmic ray station of Uppsala this is made by a precision aneroid barometer, which is photographed simultaneously with the cosmic ray recording. The mean pressure of 2-hour periods is calculated from

$$P = (P_0 + P_1 + P_2) \frac{1}{3}$$

where P_0, P_1 and P_2 are the measurements at the start, in the middle and at the end of the 2-hour period respectively.

At Murchison Bay a 24-hour barograph was used, which every third hour was calibrated by a standard mercurial barometer. The 2-hour mean was taken directly from the barograph registration. This method is better than the method used at Uppsala when the pressure variations are non-linear. However as a rule the difference between the methods are very small.

To get an estimate of the variance of $\mathcal{E}$ for pressure data we have calculated the difference in recordings from the Uppsala Cosmic Ray Station and the Meteorological Institution of Uppsala University. The two stations are lying about 3 km apart and at the same height above the sea level. We assume that the stations registrate the same pressure. The data from the Meteorological Institution is determined from a 24-hour barograph which is corrected by readings of a good mercurial barometer three times a day at 08.10, 14.10, and 20.10 L.T.. In the calculation only the values at 08, 14 and 20 L.T. have been used to get the best determined observations. The registrations are independent and we have

$$\hat{D}^2(P_C-P_M) = D^2(\epsilon_C) + D^2(\epsilon_M)$$

where C indicates the cosmic ray station and M the meteorological station. Fig. 8 shows the monthly mean and the variance of (P_C-P_M) for the year Oct. 1959 - Sept. 1960. For the whole year we have

$$D(\epsilon_C) + D(\epsilon_M) = 0.300 \pm 0.007 \text{ mb}$$

The method of recording the atmospheri. pressure is supposed to be more accurate at the meteorological station. We then estimate

$$\hat{D}(\epsilon_C) = 0.2 \text{ mb}$$

As a rule we want the mean pressure over a period e.g. 2 hours. The standard error of such a pressure data will be dependent of how many observations are made, the method used and the time variations of the pressure during the period.

Conclusions

Data uncorrected for atmospheric effects. Our calculations have given estimates of the error variances from different cosmic ray instruments. As long as we choose periods during which the instruments are working reliably, ϵ is a random variable. $D(\epsilon)$ is then the standard deviation of the ϵ-distribution at a given true counting rate N, equal the mean intensity of cosmic radiation during the period of observation By taking the ratio between the calculated standard deviation and the standard deviation estimated from the Poisson distribution we got $D(\epsilon)$ as a function of the counting rate N. For simplicity we write the moments of the ϵ-distribution in this section as $E(N)$, $D(T)$, $D^2(P)$ and so on, since the parameters N, T, P... are assumed to have only random statistical fluctuations.

The standard deviations in this paper are calculated from periods during which the instruments have been working satisfactorily. From our results an estimate of the magnitude of $D(N)$ can be made for uncorrected data from Uppsala and Murchison Bay

IGY neutron monitor: $\hat{D}(N) = 1.2\sqrt{N}$
Counter telescopes: $\hat{D}(N) = 1.15\sqrt{N}$

It has already been pointed out that these figures might differ from time to time between different instruments.

Data corrected for atmospheric effects. The registrated cosmic ray intensity at ground levels is a function of atmospheric parameters. To use these data in time series analyses they have to be corrected. In this paper we use simple linear Duperier formula and at a certain time we have

$$N_{c_1} = N_{r_1}\left[1 + \alpha(P_1-P_0) + \beta(T_1-T_0) + \gamma(H_1-H_0)\right]$$

where N_c is the corrected and N_r the registrated value; P_1, T_1 and H_1 are the observed values and P_0, T_0 and H_0 are mean values. In the following P, T and H indicates the differences between the observed values and the means. The error varaiance of N_c is

$$D^2(N_c)=D^2(N_r) + D^2(N_rP\alpha)+D^2(N_rT\beta)+D^2(N_rH\gamma)+2D^2(N_r)\,B \qquad (20)$$

where

$$B = E(\Psi)E(P)\left[1+E(\beta)E(T) + E(\gamma)E(H)\right] + E(\beta)E(T)\left[1+E(\gamma)\,E(H)\right] + E(\gamma)E(H)$$

and the variance of a product abc is

$$D^2(abc)=E(bc)D^2(a) + E(ac)D^2(b)+E(ab)D^2(c)+D^2(a)D^2(b)D^2(c)$$

We have assumed all variables as independent. This is not exactly true for P and H but their covariance term may be neglected. In the first approximation we may also neglect $D^2(\alpha)$, $D^2(\beta)$, $D^2(\gamma)$ and their products. We further use the following error variances and constants:

$\hat{D}^2(N_r) = 1.44\ \hat{N}_r$ IGY neutron monitor (from this paper)

$\hat{D}^2(N_r) = 1.30\ \hat{N}_r$ Counter telescopes (-"-)

$\hat{D}^2(P) = 0.04\ mb^2$ (from Trefall, Nordö, 1959)

$\hat{D}^2(T) = 5(^\circ C)^2$ (-"-)

$\hat{D}^2(H) = 25 \times 10^{-4}\ km^2$ (-"-)

$\hat{\alpha} = -0.73$ per cent/mb IGY neutron monitor

$\hat{\beta} \approx \hat{\gamma} \approx 0$ -"-

$\hat{\beta} = +0.1$ per cent/$^\circ$C Counter telescopes

$\hat{\gamma} = -6$ per cent/km -"-

For long periods when we can assume

$P = T = H = 0$

the error variance of corrected value is

IGY neutron monitor: $\hat{D}^2(N_c) = 1.44\ \hat{N}_r + 2 \times 10^{-6}\ \hat{N}_r^2$ (21)

Counter telescopes: $\hat{D}^2(N_c) = 1.30\ \hat{N}_r + 14.1 \times 10^{-6}\ \hat{N}_r^2$ (22)

As seen from eq.(21) this is not valid when P, T and H differ from zero. $D(N_c)$ is dependent of the true observed value of the atmospheric parameters. This is important when we want to calculate the standard errors of short period data e.g. 2-hour values. Eq. (21) will then give the error variance. However, eq.(20) and (22) will in many cases give a good estimate. The time variations of the atmospheric parameters will affect the standard errors of the corrected data according to during which period the means of the parameters are calculated.

Remarks: It must be pointed out the importance of estimating the standard errors of data with care due to which kind of analysis they will be used for. For fine structure interpreting it might sometimes be necessary to make careful calculations but on the other side rough estimates are often satisfying. The labour of the error calculations must be weighted against the need of accurate estimations. However, it must be stressed that the use of the Poisson distribution gives an underestimation of the standard errors of cosmic ray data.

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The author is very grateful to Dr N.R. Parsons at the University of Tasmania for delivering neutron monitor data from Mt. Wellington and to Professor G.H. Liljequist, Uppsala, for the pressure data from the Meteorological Institute of the University of Uppsala.

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Errata

In Fig. 8 is written $S(P_C - P_M)$. Read $\hat{D}(P_C - P_M)$.

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Table 1

Neutron monitor

Station	Period	Scaling factor m	$\hat{M}$	n	$S=\hat{D}(x-y)$ acc. to eq.(11)	Overestimate in per cent	$\hat{S}(S)$	$S_p=\hat{D}_p(x-y)$ acc. to Poisson eq.(2)	$\hat{S}(S_p)$	$\frac{S}{S_p}$	$\hat{S}(\frac{S}{S_p})$
Murchison Bay	11/9 57-10/12 58	1	+29.35	2175	190.27	$\ll 1$	2.88	161.45	2.45	1.179	0.025
Murchison Bay	11/12 57-10/6 58	1	-29.75	4305	182.20	$\ll 1$	1.96	155.94	1.68	1.168	0.018
Uppsala	1/8 57-30/2 58	1	+94.47	4907	188.20	< 1	1.90	157.76	1.59	1.193	0.017
Mt.Wellington	11/9 57-20/1 58	64	+479.71	3018	239.98	> 1	3.06	190.14	2.45	1.262	0.026

Ratio

Station	Period	Scaling factor m	$\hat{k}$	n	$S=\hat{D}(\frac{x}{y})$ acc. to eq.(13)	$\hat{S}(S)$	$S_p=\hat{D}(\frac{x}{y})$ acc. to Poisson eq.(2)	$\hat{S}(S_p)$	$\frac{S}{S_p}$	$\hat{S}(\frac{S}{S_p})$
Mt.Wellington	11/9 57-20/1 58	64	0.9739	3013	0.012836	0.000165	0.010242	0.000132	1.253	0.023

Table 2 **Duplex cubical counter telescope**

Station	Period	Scaling factor	$\hat{M}x10^2$	r	$S=\hat{D}(x-y)x10^2$ acc. to eq. (11)	Overestimate in per cent	$\hat{S}(S)$	$S=\hat{D}_p(x-y)x10^2$ acc. to Poisson eq. (2)	$\hat{S}(S_p)$	$\frac{S}{S_p}$	$\hat{S}(\frac{S}{S_p})$
Uppsala	25/12 58-11/2 59	100	-133	560	527.2	< 1	15.7	433.8	12.9	1.218	0.051
	26/6 - 26/8 59	100	-726	747	458.2	< 1	11.8	428.2	11.1	1.069	0.036
	9/9 - 4/10 59	100	-623	308	464.8	< 1	18.7	432.7	17.4	1.074	0.061
Murchison Bay	8/11 57-5/1 58	100	-726	670	576.8	< 1	15.5	445.6	12.2	1.294	0.049
	17/1 - 2/4 58	100	-728	835	554.7	< 1	13.6	431.4	10.8	1.286	0.044
	6/9 - 26/9 57	100	-544	330	538.9	< 1	21.0	442.0	17.2	1.219	0.067

Table 3

Directional telescope

a 12/8-1/11 1957
b 5/11-9/12 1957
c 25/12-9/2 1957-58
d 2/3-2/4 1958
e 10/6-27/9 1958
f 21/8-15/10 1957
g 25/12-25/1 1957-58
h 31/7-1/9 1958
i 14/9-10/10 1957
j 29/12-21/2 1957-58
k 12/5-14/6 1958
l 13/9-10/10 1957
m 4/1-28/2 1958
n 18/4-17/5 1958

Station	Channels	Period	M	n	$S=\hat{D}(x-y)$ acc. to eq. (11)	Over-estimate in per cent	$\hat{S}(S)$	$S_p=\hat{D}_p(x-y)$ acc. to eq. (2)	$\hat{S}(S_p)$	$\frac{S}{S_p}$	$\hat{S}(\frac{S}{S_p})$
Uppsala	3-1	a	+460	951	259.2	~1	5.9	240.7	5.5	1.077	0.035
		b	+607	418	265.8	~1	9.2	238.9	8.3	1.113	0.054
		c	+600	532	289.7	~1	8.9	242.9	7.5	1.193	0.052
		d	+581	356	298.5	~1	11.2	241.1	9.0	1.238	0.066
		e	+551	1252	252.2	~1	5.0	239.6	4.8	1.053	0.030
	7-5	a	+596	974	291.1	~1	6.6	239.6	5.5	1.215	0.039
		b	+563	418	237.1	~1	8.2	238.5	8.3	0.994	0.050
		c	+555	563	239.0	~1	7.1	241.1	7.2	0.991	0.042
	4-2	f	-483	668	246.5	~1	6.7	238.4	6.5	1.034	0.039
		b	-350	419	256.4	< 1	8.8	236.8	8.2	1.083	0.052
		g	-514	381	313.8	~1	11.4	241.2	8.6	1.301	0.067
		h	-637	363	249.7	~1	9.2	239.6	8.9	1.042	0.055
	8-6	f	+778	669	259.1	> 1	7.1	240.0	6.6	1.080	0.042
		b	+481	419	262.1	~1	9.0	238.2	8.3	1.100	0.053
		g	+490	378	255.7	~1	9.3	243.4	8.8	1.051	0.055
		h	-359	391	269.7	< 1	9.6	237.6	8.5	1.136	0.057
Murchison Bay	3-1	i	-572	311	294.6	~1	11.8	241.0	9.4	1.222	0.066
		j	-299	636	257.2	< 1	7.2	245.0	6.8	1.050	0.041
		k	-141	396	271.7	< 1	9.7	240.7	8.6	1.129	0.056
	7-5	i	+297	324	256.7	< 1	10.0	241.8	9.5	1.062	0.059
		j	+253	675	312.3	< 1	8.8	245.3	6.8	1.273	0.051
	4-2	l	-665	316	290.2	~1	11.5	242.0	9.3	1.199	0.067
		m	-621	637	294.4	~1	8.2	245.5	6.7	1.190	0.047
	8-6	l	+160	322	291.3	< 1	11.5	244.1	9.4	1.193	0.066
		m	+170	643	280.5	< 1	7.8	243.6	6.5	1.151	0.045
		n	+280	354	286.2	< 1	10.7	239.2	8.9	1.196	0.063

Table 4 a

Directional telescope at Uppsala

a 12/8 - 1/11 1957
b 5/11 - 9/12 1957
c 25/12- 9/2 1957-58
d 12/3 - 2/4 1958
e 10/6 - 27/9 1958
f 21/8 - 15/10 1957
g 5/11 - 9/12 1957
h 25/12- 25/1 1957-58
i 31/7 - 1/9 1958

Channel	Period	$S^2 = \hat{D}^2(\varepsilon)$	$S_p^2 = \hat{D}_p^2(\varepsilon)$ acc. to eq. (2)	$\frac{S}{S_p}$	$\hat{S}(\frac{S}{S_p})$
1	a	32288	28747	1.16	0.03
	b	36560	28226	1.14	0.05
	c	33628	29221	1.07	0.05
	d	32329	28729	1.06	0.05
	e	35858	28426	1.12	0.03
2	f	31741	28603	1.05	0.04
	g	41529	28203	1.21	0.06
	h	55975	29354	1.38	0.07
	i	35063	29012	1.10	0.05
3	a	31649	29206	1.04	0.03
	b	35970	28835	1.12	0.05
	c	53716	29803	1.34	0.06
	d	56751	29389	1.39	0.07
	e	27737	29000	1.00	0.03
4	f	32391	28234	1 07	0.04
	g	23360	27846	0.93	0.05
	h	41147	28833	1.20	0.06
	i	28568	28377	1.00	0.05
5	a	39939	28411	1.19	0.04
	b	30186	28153	1.03	0.04
	c	31317	29226	1.03	0.04
6	f	33302	28407	1.08	0.04
	g	32774	28122	1.08	0.05
	h	27679	29340	1.08	0.05
	i	32614	28235	1.08	0.05
7	a	41574	29022	1.20	0.04
	b	27629	28714	0.98	0.05
	c	29236	29784	0.99	0.04
	d	30082	28735	1.02	0.05
	e	35277	28447	1.11	0.03
8	f	37382	29185	1.12	0.04
	g	35408	28605	1.11	0.05
	h	36358	29904	1.10	0.05
	i	41392	28217	1.21	0.06

Table 4 b

Directional telescope at Murchison Bay

a 14/9 - 10/10 1957
b 29/12 - 21/2 1957-58
c 12/5 - 14/6 1958
d 13/9 - 10/10 1957
e 4/1 - 28/2 1958
f 18/4 - 17/5 1958

Channel	Period	$S^2 = \hat{D}^2(\varepsilon)$	$S_p^2 = \hat{D}_p^2(\varepsilon)$ acc. to eq. (2)	$\frac{S}{S_p}$	$\hat{S}(\frac{S}{S_p})$
1	a	35301	29336	1.10	0.06
	b	31752	30301	1.02	0.04
	c	32004	29211	1.05	0.05
2	d	35811	29622	1.10	0.06
	e	55285	30452	1.35	0.05
	f	40575	29404	1.18	0.06
3	a	50594	28750	1.33	0.07
	b	34683	29797	1.08	0.04
	c	41812	28711	1.21	0.06
4	d	55182	28942	1.38	0.08
	e	32515	29840	1.04	0.04
5	a	36538	29055	1.12	0.06
	b	40654	29955	1.16	0.05
	c	47283	28856	1.28	0.06
6	d	44431	28716	1.24	0.07
	e	36245	29581	1.11	0.04
	f	45925	28485	1.27	0.07
7	a	28470	29388	0.99	0.05
	b	56978	30198	1.37	0.05
8	d	47228	28870	1.28	0.07
	e	49124	29751	1.29	0.05
	f	36011	28772	1.12	0.06

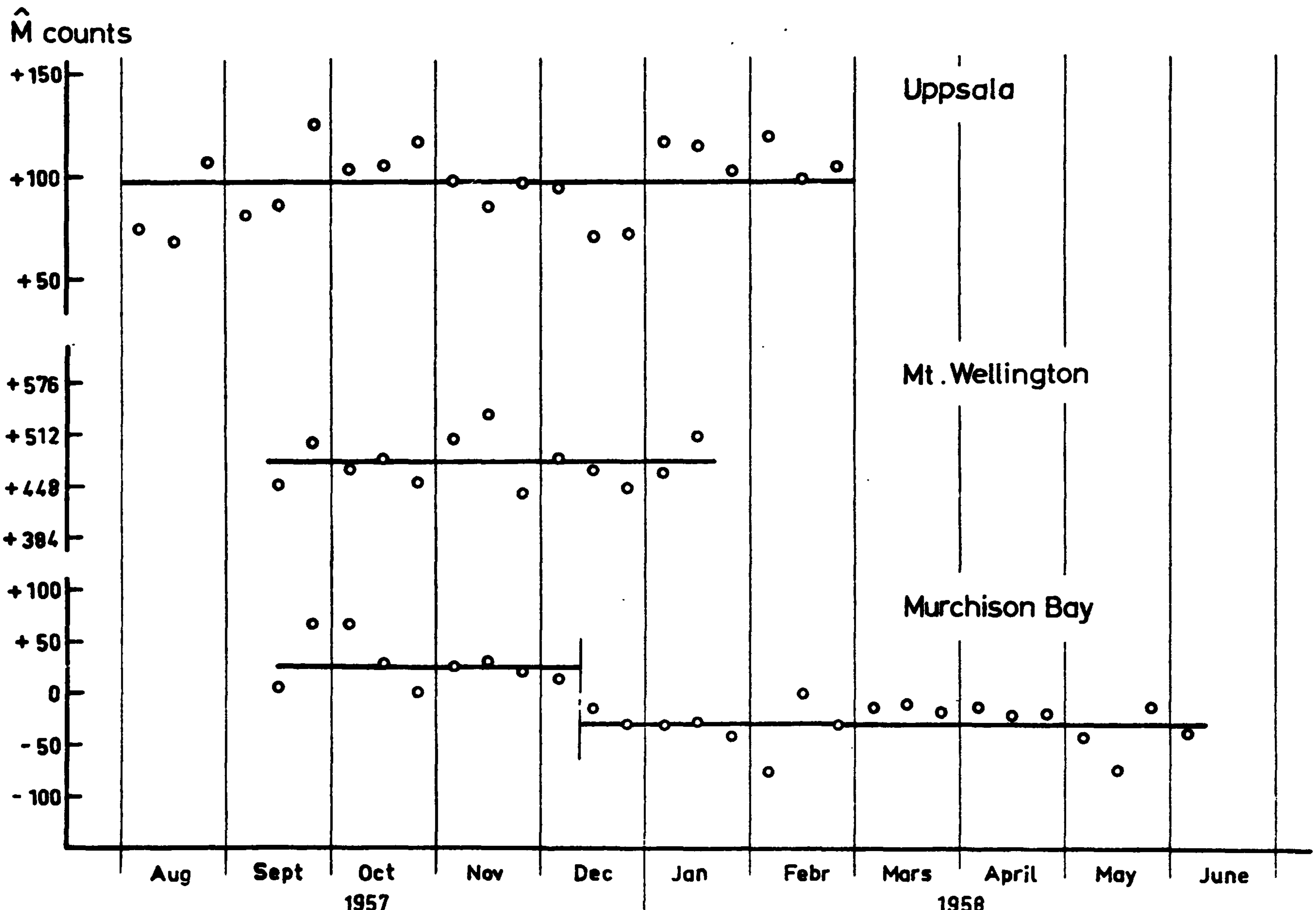

Fig.1. 10-day means of (x-y) from the neutron monitors.

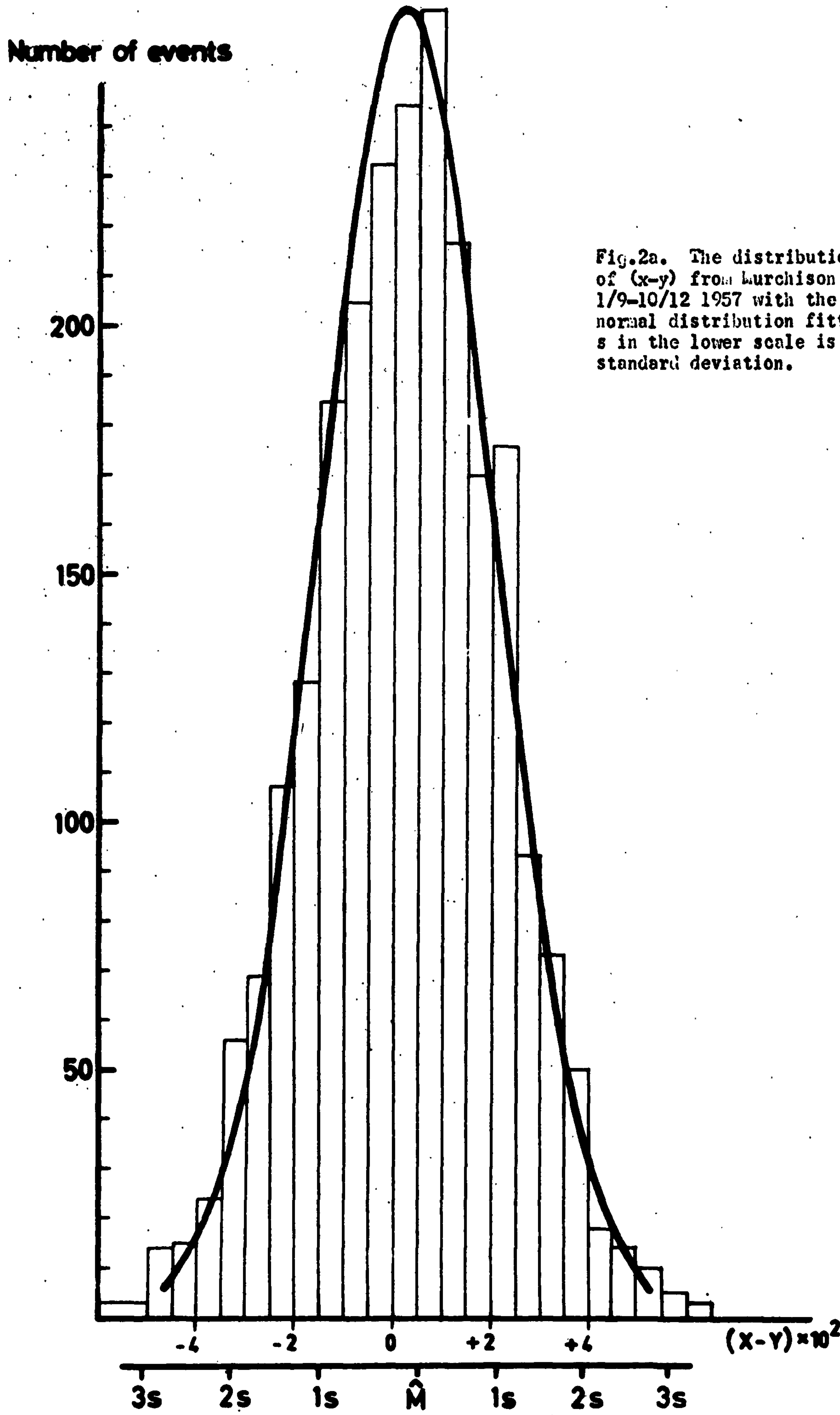

Fig.2a. The distribution of (x-y) from Murchison Bay 1/9-10/12 1957 with the normal distribution fitted. s in the lower scale is the standard deviation.

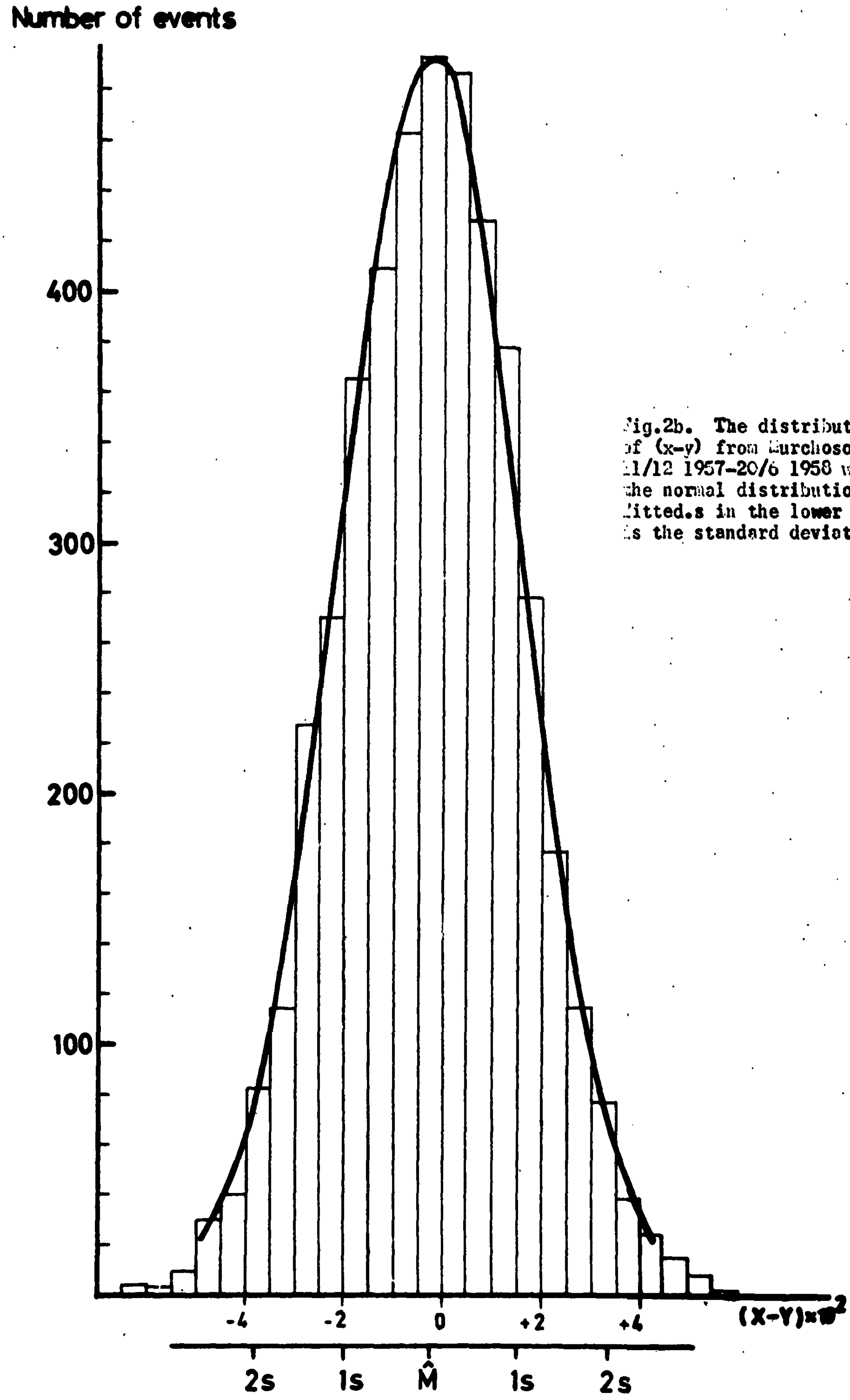

Fig.2b. The distribution of (x-y) from Murchoson Bay 11/12 1957-20/6 1958 with the normal distribution fitted. s in the lower scale is the standard deviation.

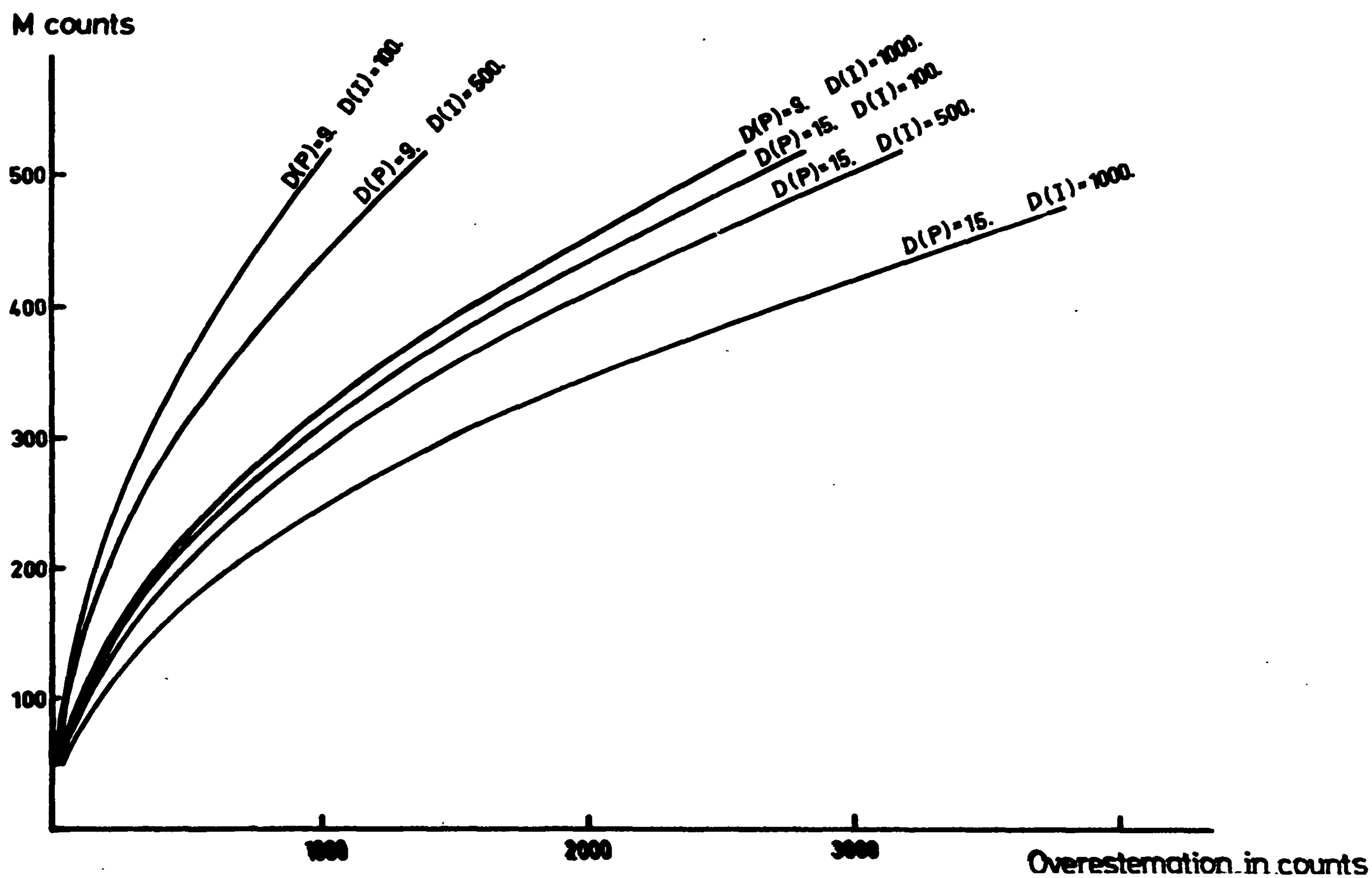

Fig.3. The overestimation of $D^2(\varepsilon_x) + D^2(\varepsilon_y)$ when using eq.(11) as a function of M for some values of D(P) and D(I).

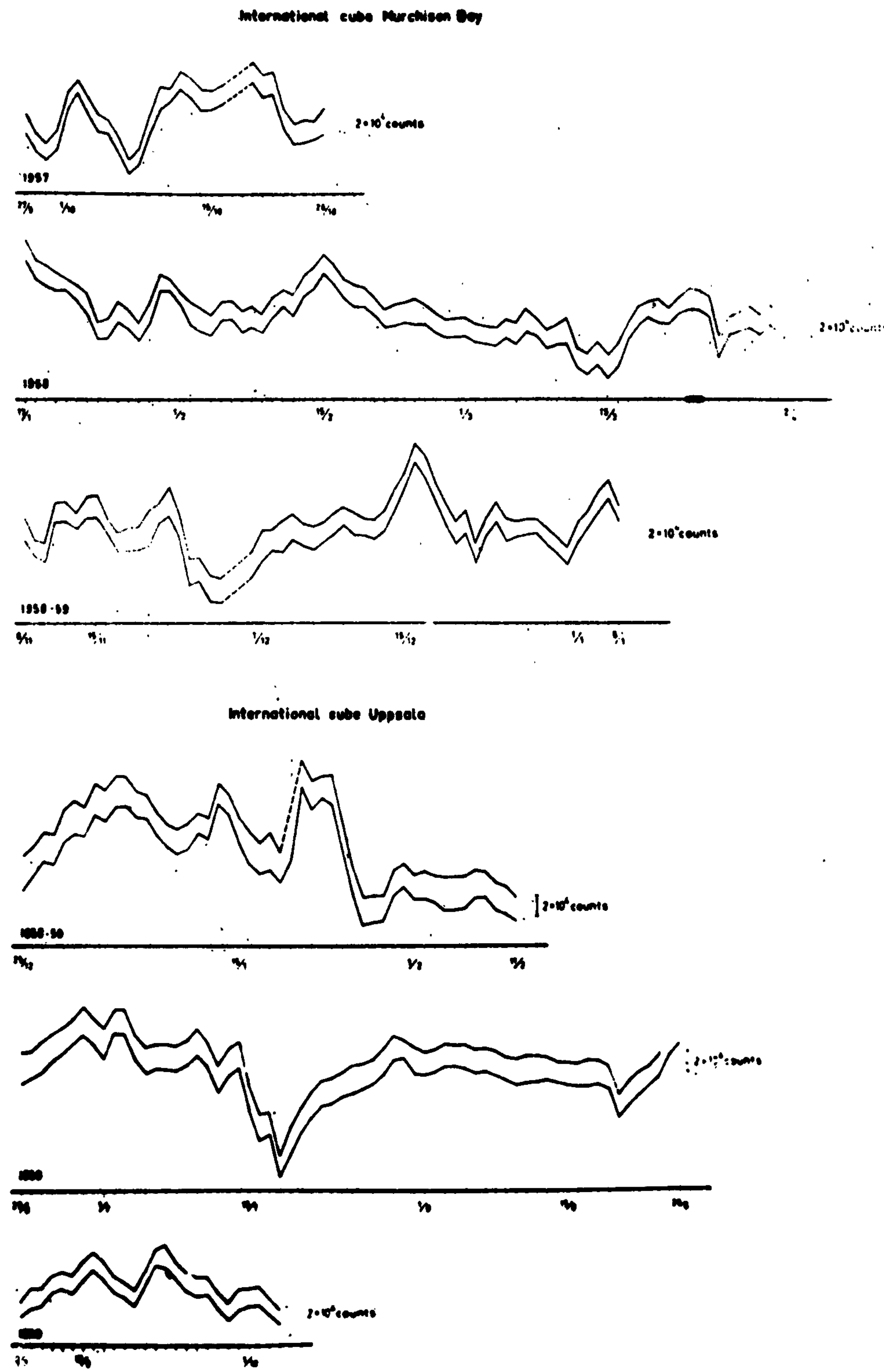

Fig. 4. Daily mean intensity for the dublex cubical counter telescopes at Uppsala and Murchison Bay.

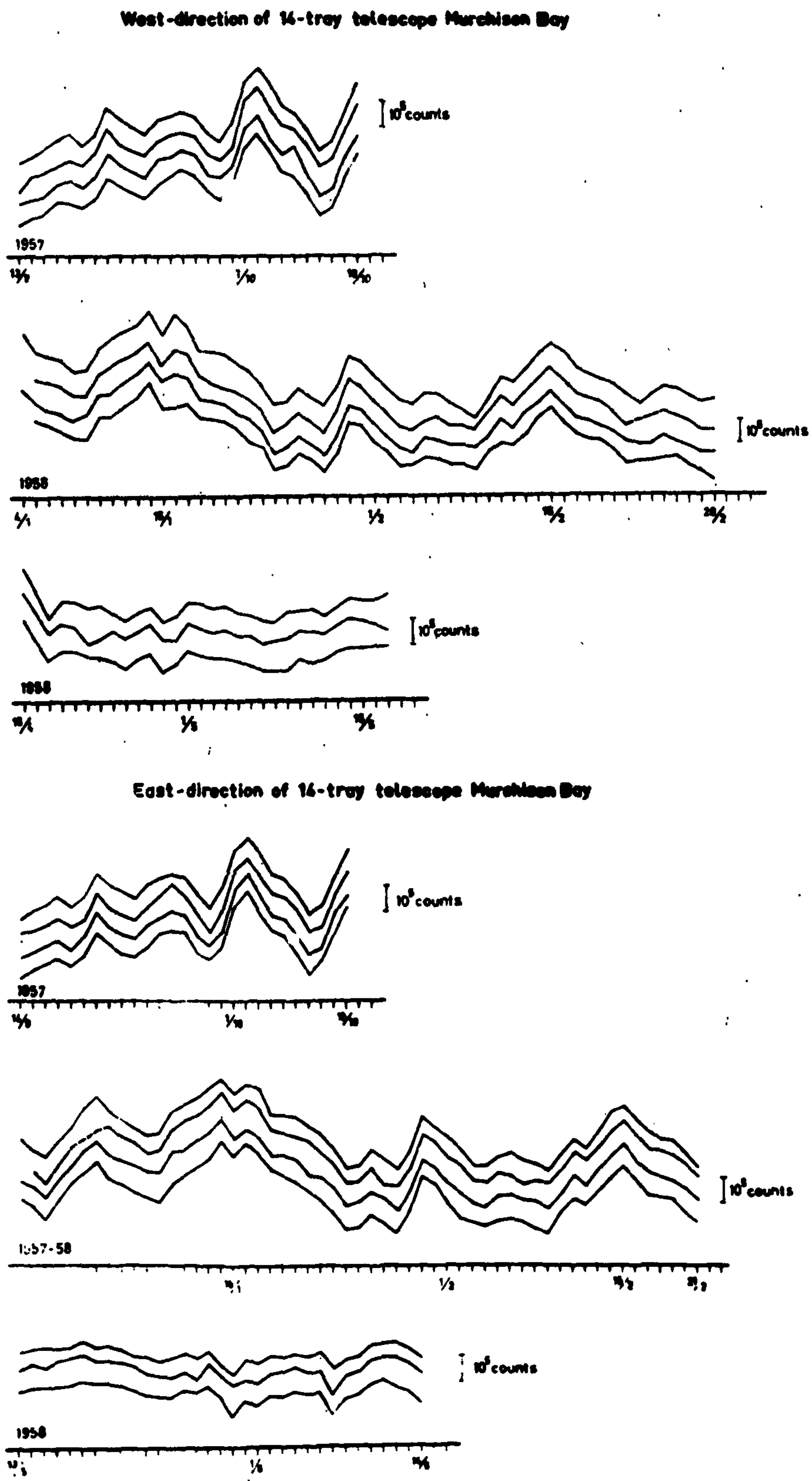

Fig.5. Daily mean intensity for the directional telescope at Murchison Bay

West-direction of 14-tray telescope Uppsala

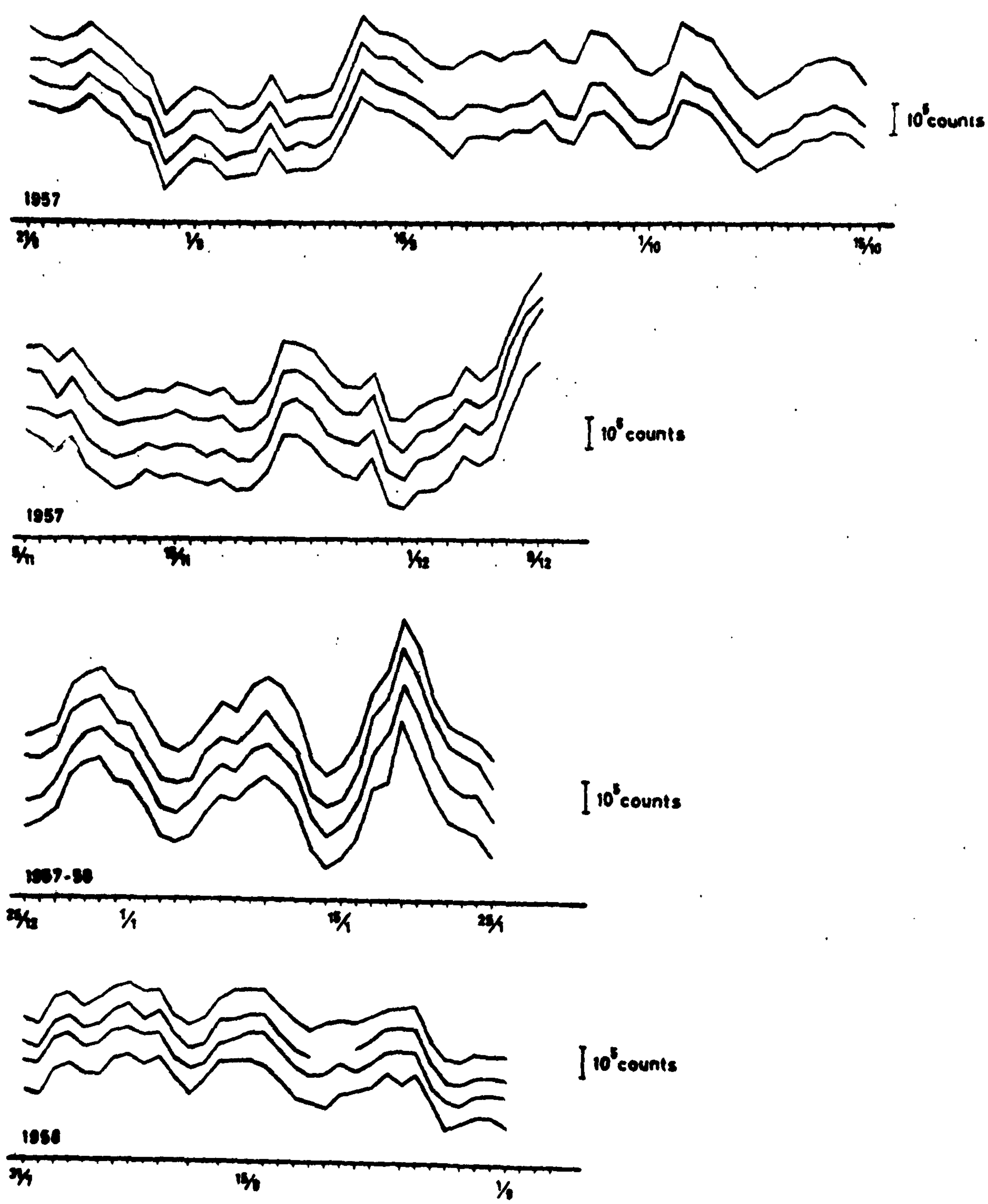

Fig.6. Daily mean intensity for the directional telescope at Uppsala.

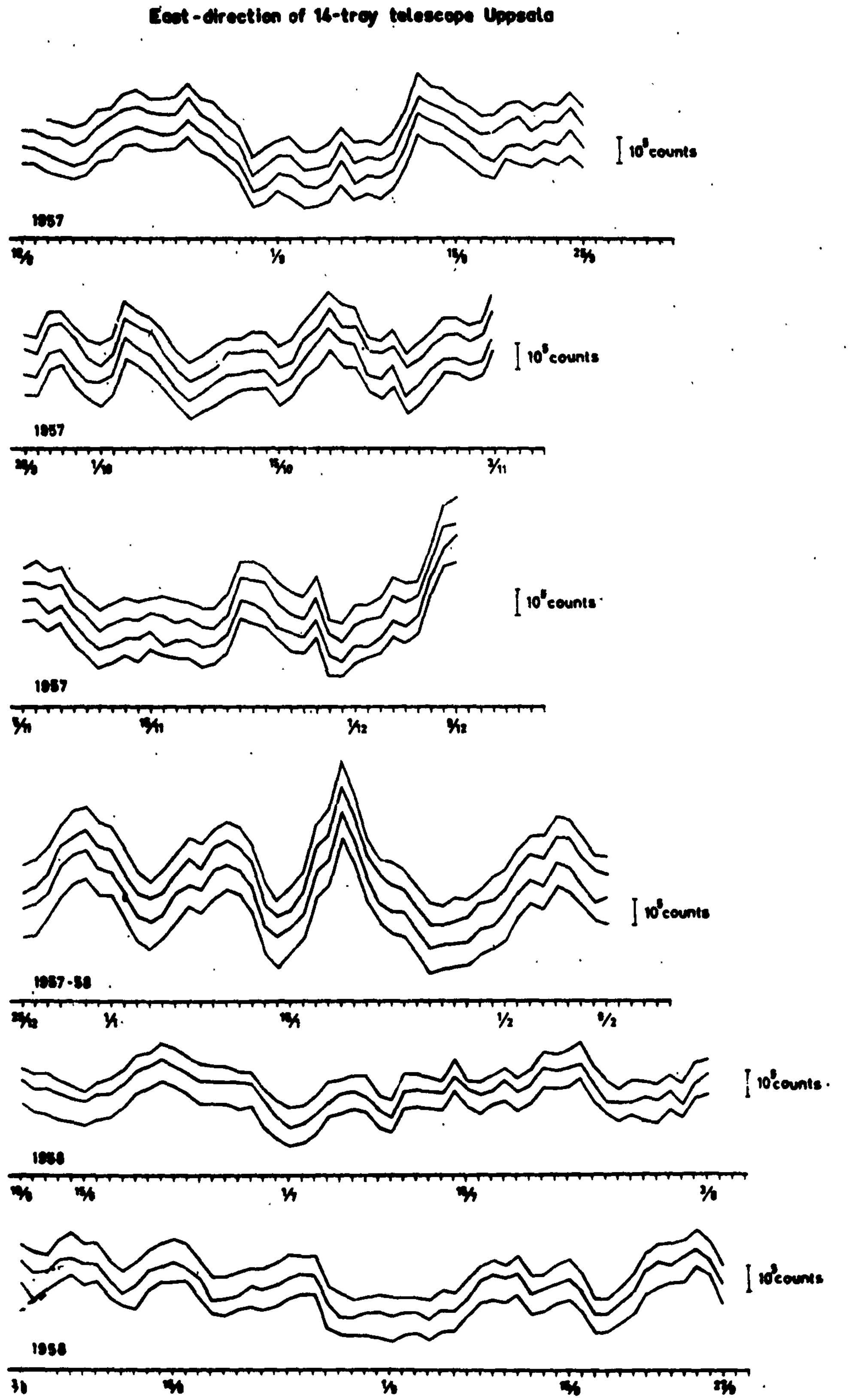

Fig.7. Daily mean intensity for the directional telescope at Uppsala.

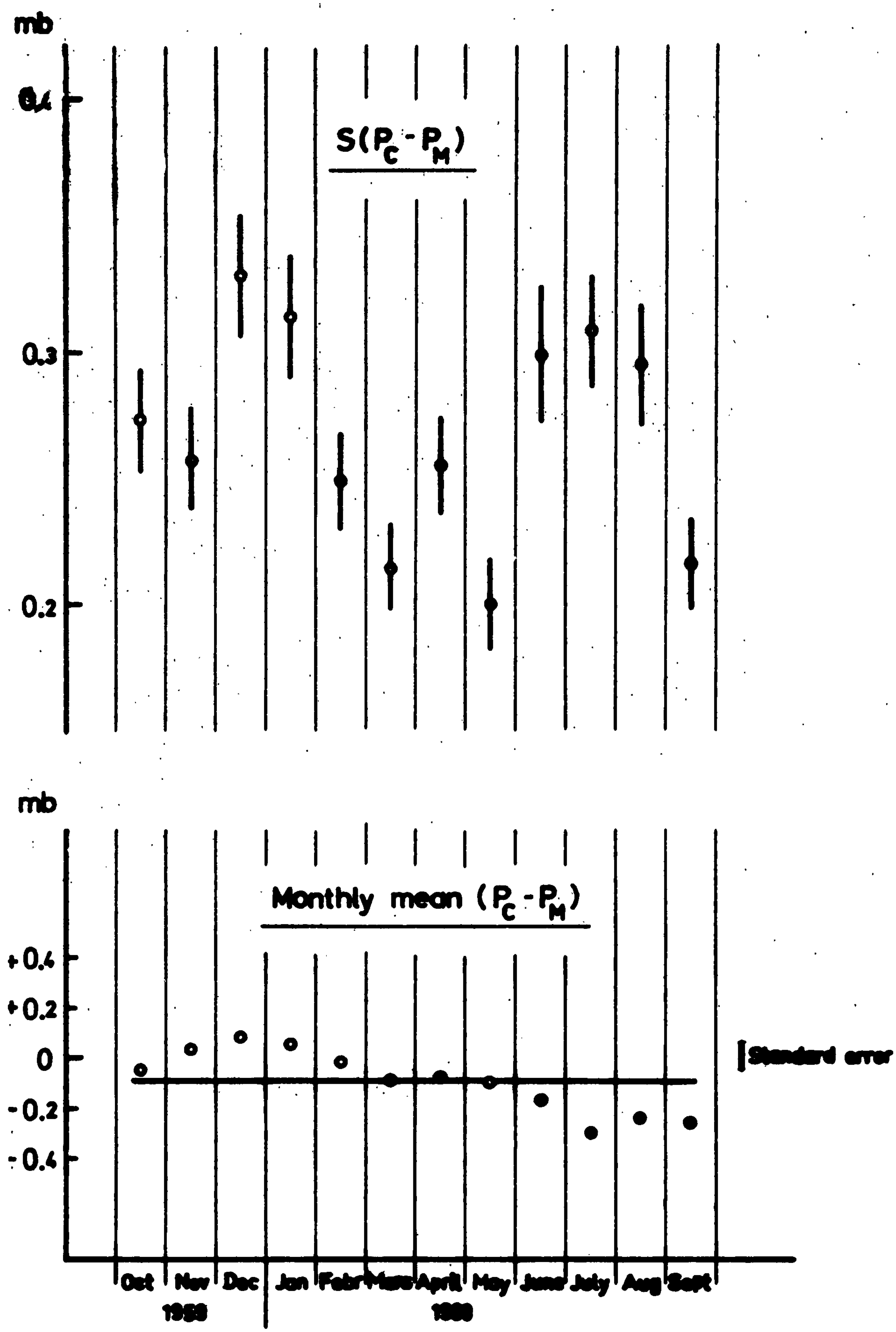

Fig.8. The variation of $\hat{E}$ (P_C-P_M) and S (P_C-P_M) Oct.1959–Sept.1960

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